Assessment and Selection of Stress Field Models

Re – Entrant Corners or Dapped End Beam Models

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ABSTRACT

Over the years, due to the diverse needs and structural challenges, more and more studies are being carried out to better understand the behavior of structures. In a structure, there may be regions called D – regions, zones of static or geometric discontinuity, in which Bernoulli's hypotheses are not applicable. Due to its particularity, several methods, models, calculation forms and simplifications have been developed that allow a better design of these regions, as well as a better simulation of their performance.

The strut and tie models and the stress field models have been one of the main tools used in these situations. However, they have limitations regarding the ductility and the performance of the structure under service conditions. Thus, they leave several questions open regarding the choice of the most suitable models to be used.

Thus, in this work, the D regions of a dapped end beam, simply supported and subjected to a concentrated load in the middle span, are studied. Taking into account the path of the load inside the structure, different levels of force redistribution were considered, from which different strut and tie models and the respective reinforcement were developed.

The service behavior of the different models was evaluated and studied through a non-linear analysis of finite elements.

Finally, through this dissertation, it is possible to draw conclusions regarding the behavior of these regions and also obtain information, parameters and references, which can be of great use for the design of this type of structure.

Key – words: discontinuity regions, strut-and-tie models, stress field models, finite element method, dapped end beam, non-linear analysis

1. INTRODUCTION

Understanding the behavior of a structure given certain conditions is extremely important for structural design. Over time and technological developments, discoveries have been made regarding the performance of reinforced concrete elements. As a result of several researches and studies carried out, the classic truss model is introduced by Wilhelm Ritter at the end of the 19th century, more specifically in 1899, which was later refined by Emil Morsch in 1912.

However, this model naturally presented some limitations, as it may only be applied at certain areas of a structure, called B-regions (regions in which Bernoulli's hypothesis is applicable). Thus, there is a need to improve and refine these models, giving rise to the strut and tie models.

The strut and tie models, besides being able to be applied in the aforementioned B-regions, expand their use to D-regions, which are areas of static or geometric discontinuity (areas in which Bernoulli's hypothesis is not applicable).

Despite the advantages of using the strut and tie models, it turns out that, for a given situation, there may be a large number of models that satisfy the balance of internal forces, thus leaving a wide margin of choice for suitable models open. Due to the need to overcome the limitations mentioned, several methods of automatic model selection have been developed over the years, based on the most varied criteria. One of these methods is the finite element method (FEM) developed by Ruiz and Muttoni in 2007.

Dapped end beams, the object of study in this work, contain discontinuity regions, so it is necessary to carefully study their behavior in the different loading phases.

Thus, the main objective of this dissertation is to evaluate the behavior in service of dapped end beams/re-entrant corners subjected to a concentrated load in the middle span and to establish criteria for the selection of strut and tie models that help to reduce the degree of uncertainty of the designer in the design of this type of structures.

The evaluation of this type of models was carried out by establishing different distributions of the internal forces, obtaining the respective reinforcement details and analyzing the system's response to the indicated load. The simulation and analysis of the different models was done using a finite element program (EvalS), by Miguel Ferreira (2017), with the aid of spreadsheet programs developed for the interpretation and graphical representation of the results.

2. DISCONTINUITY REGIONS

D-regions, also known as discontinuity zones, are regions where the strain distribution is non – linear. These usually arise due to geometric discontinuities (corners, bends, openings) or due to static discontinuities (such as concentrated loads). The following figure [Figure 1] illustrates the stresses in this type of zones.

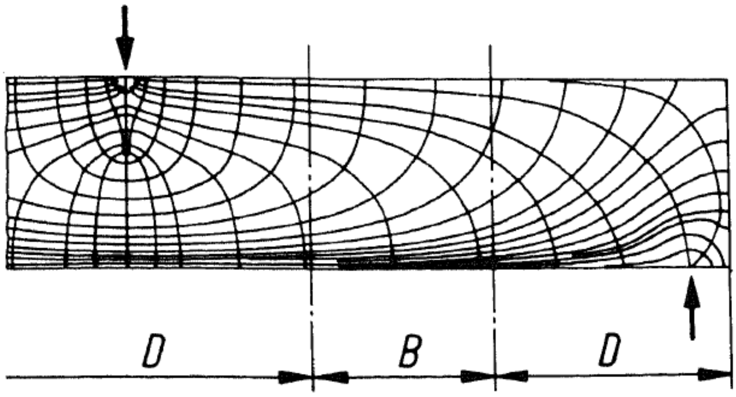

Figure 1 – Stress trajectories in a B-region and near discontinuities (D-regions) (Schlaich, Schafer, & Jennewein, 1987).

3. MODELING PRINCIPLES

As already mentioned, the dimensioning of discontinuity zones can be performed using strut and tie models as a resource. These models are made up of: struts that represent compression stress fields; ties that represent the reinforcement of the tensile zones; and the connection nodes that are stress deviation zones.

First, for the application of these models, the forces applied at the borders of the discontinuity regions, which can be external forces or internal forces from adjacent regions, must be known and equilibrated.

Then, to elaborate the strut and tie model, the load path method can be used, the struts and tie are calculated based on equilibrium conditions. Finally, the details of the reinforcement must be appropriate to the forces and location of the ties, and the minimum amounts of reinforcement must be placed in the areas where no ties have been assigned.

4. LOAD PATH METHOD

The load path can be designed by establishing the trajectories that direct the forces from their point of application to the support. After forces at the borders are known, their external equilibrium must be established.

The load paths begin and end at the center of gravity of the corresponding stress diagrams and tend to follow the shortest possible path.

In the tracing of the load trajectories, certain deviations and curvatures may arise that generate stresses with different directions, the paths must be replaced by polygons and considered struts and ties to ensure balance in the nodes, as shown in the following figure [Figure 2].

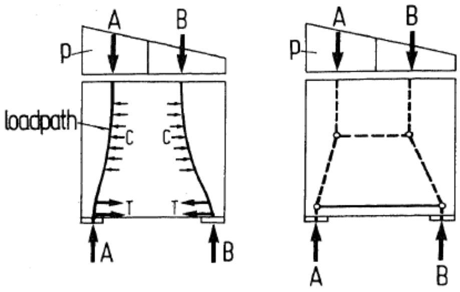

Figure 2 – Load paths and corresponding strut and tie model (Schlaich, Schafer, & Jennewein, 1987).

5. MODELS SELECTION METHODS

Because there is a certain freedom in creating models, there is not only one and great solution. However, Schlaich, Schaefer & Jennewein in 1987, suggest that forces tend to follow paths that generate less internal forces and deformations. Based on this, they presented an equation that derives from the principle of minimum strain energy and that can be used in the selection and optimization of models:

$$\Sigma F_i l_i \varepsilon_{mi} = Minimum$$

Onde:

F_i – force in strut or tie i;

l_i – length of member i;

ε_{mi} – mean strain of member i;

6. FINITE ELEMENT MODELING AND ANALYSIS

As already mentioned, in this dissertation the finite element method and the EvalS program developed by Miguel Ferreira (2017) were used as resources. The FEM consists of a non-linear analysis and was developed by Ruiz and Muttoni in 2007.

For its application, it is necessary to know only a limited number of parameters, such as strength and modulus of elasticity.

6.1 Formulation

The behavior of the concrete is simulated by neglecting its tensile strength and assuming that the principal stress directions are parallel to the principal strain directions, the stress value is also obtained through these strains. The compressive strength value of the concrete must be corrected according to the transversal tension extension. The concrete is considered to have an elastic response until yield and perfectly plastic afterwards, as can be seen in Figure 3-e).

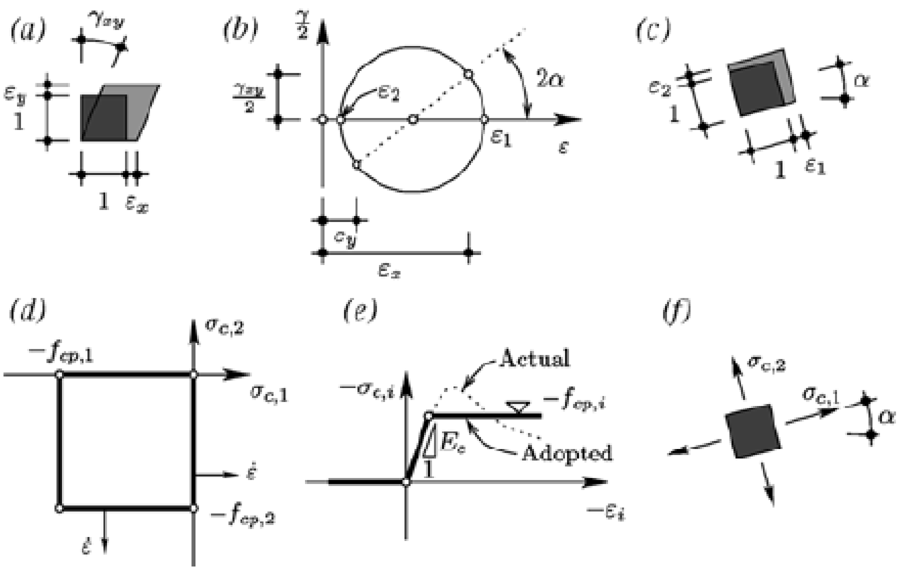

Figure 3 – Concrete modeling: a) Strains; b) Mohr´s circle and principal strains; c) Directions of principal strains; d) Yield surface for plane stress; e) Adopted stress – strain response; f) Assumed directions for principal stresses (Ruiz, Muttoni, 2007).

The simulation of concrete and its behavior is performed using CST Elements (Constant Strain Triangle) that represent triangles that have a constant deformation in their domain [Figure 4].

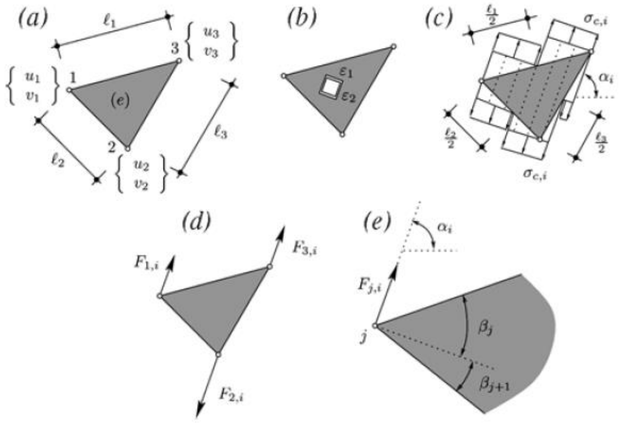

Figure 4 - CST element (constant strain triangle): a) Displacement field in element; b) Strain field in element; c) Assumed stress field for principal stress i; d) Nodal forces; e) Angle β at each node j (Ruiz, Muttoni, 2007).

The modeling of the steel is done through bar-type elements and considering that it has a uniaxial response with a bilinear elasto-plastic law, as can be seen in Figure 5. The response of the steel is defined by the resistance of the material f_y, its elastic modulus E_s, and its hardening modulus E_h.

The effects of tension stiffening, which is the increase of the axial rigidity of the tie due to the surrounding concrete, were also considered.

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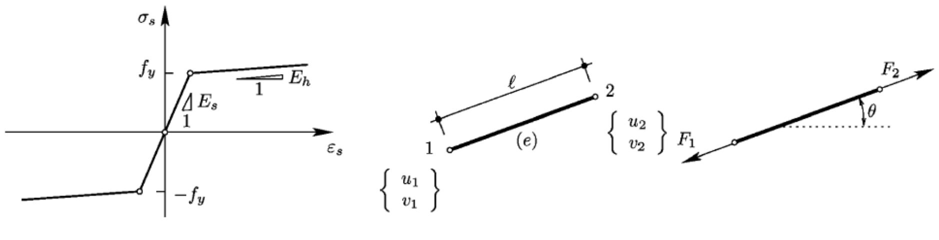

a) **b)** **c)**

Figure 5 – Steel modeling: a) Elasto – plastic behavior of steel with deformation hardening; b) Displacement field at element; c) Nodal forces at element (Ruiz, Muttoni, 2007).

The set of nonlinear equations is solved using the Newton Raphson convergence method, in which the stiffness matrix remains constant throughout the iterations.

6.2 Case Study

In this chapter, the different data and characteristics necessary to assess the behavior of the dapped end beam in study are listed. This beam has a cross-section of 0.25m x 1.0m and a span of 5m. The re-entrant corner has a cross-section with 0.25m x 0.50m and a length of 0.5m. Regarding the characteristics of the materials, C60 concrete and A500 steel were adopted.

The beam is subjected to a concentrated load of 1000kN applied to the middle span of the structure. In Figure 6, a representation of the beam is made with the characteristics and conditions stated.

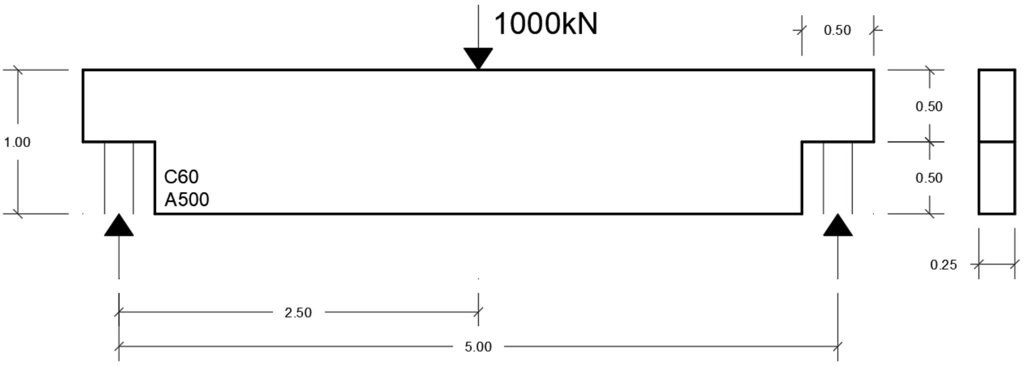

Figure 6 – Dimensions, characteristics and loading of the dapped end beam.

The model of these beams results from an overlap of two configurations, one orthogonal and the other inclined. For the construction of the strut and tie models, it was considered an angle Θ_1 of 45º and an angle Θ of 30º. This can be seen in the following figure [Figure 7].

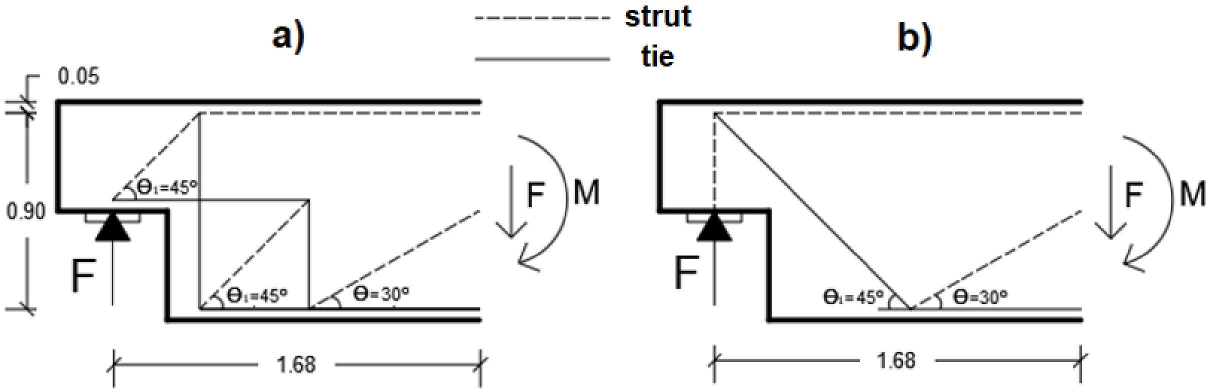

Figure 7 – Characteristics of the configuration: a) Orthogonal; b) Diagonal.

6.3 Analysis of dapped end beam models

For these analysis, five models with different characteristics were considered, assuming different load distributions, using as a resource the adoption of different values (0.0, 0.25, 0.5, 0.75 and 1.0) for the k_d variable. This variable represents the percentage of the load that is carried by the orthogonal reinforcement. Then, considering the regulations, rules and norms ruling, the forces of the STM elements and the respective reinforcements were calculated.

Subsequently, the behavior of the beam was simulated and analyzed using finite elements in the EvalS program, taking into account the established conditions and the different designs. As a result of these simulations, the stresses in the reinforcements, the compressive stresses in the concrete and the reduction of the strength of the concrete are obtained.

6.4 General assessment of model behavior

Through the construction of the studied models, it is possible to study the behavior in service of a structure for different reinforcement distributions, in order to know what the freedom in choosing the design model. This type of study is extremely important, as the behavior in service is directly linked to cracking. Thus, in Figures 8 and 9, graphs are shown with the level of stresses at each load step for each model, in order to make a general analysis.

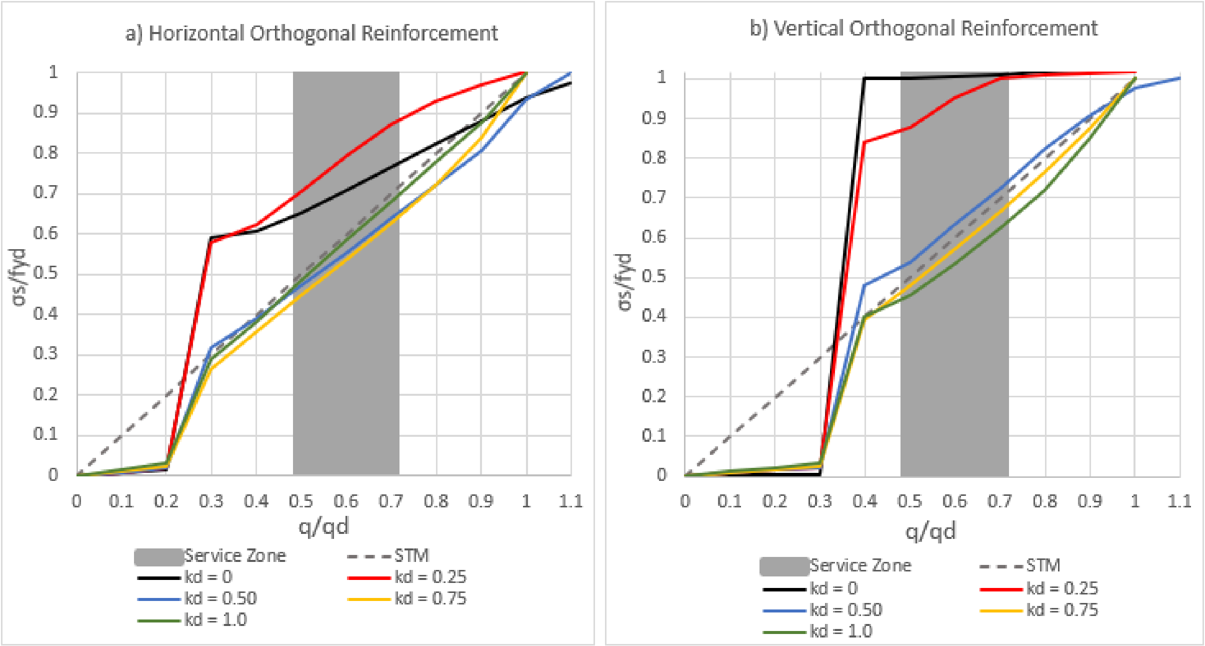

Figure 8 - Stress levels at each load step for the orthogonal reinforcement: a) horizontal; b) vertical

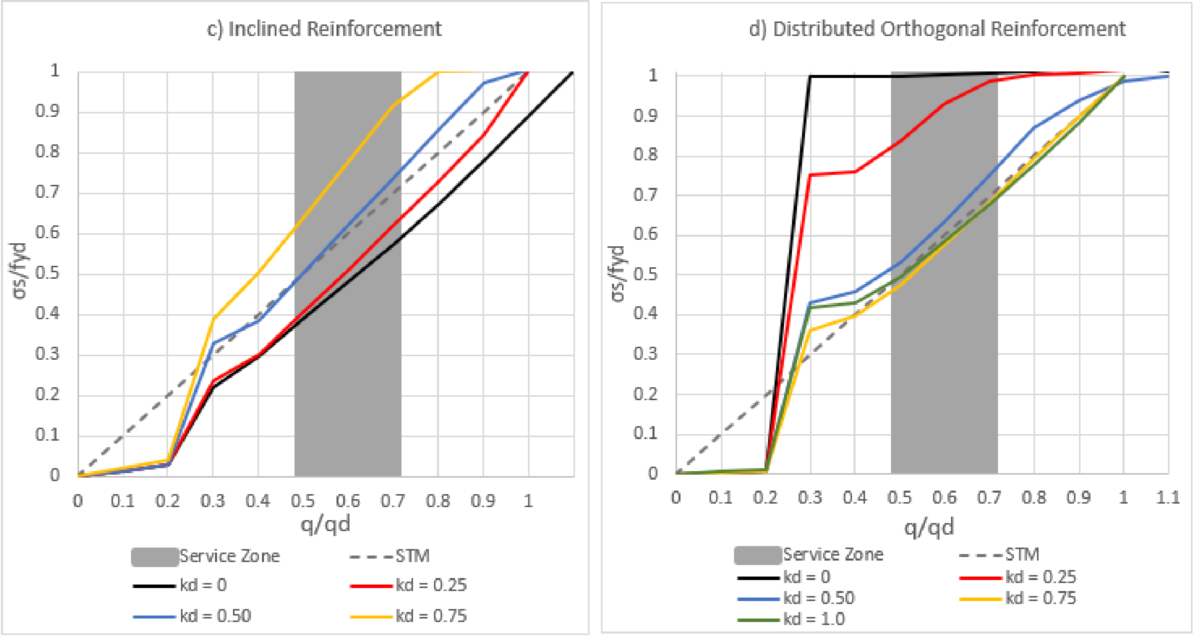

Figure 9 - Stress levels at each load step for the reinforcement: c) inclined; d) distributed.

By analyzing the graphs, it appears that the model k_d = 0.5 (half of the load balanced by orthogonal reinforcement and the other half by inclined reinforcement) presents acceptable stresses in all ties, indicating that it is the model that shows the best behavior in service .

The reference model, based on the elastic trajectories, which indicates that 75% of the load must be transmitted by the inclined reinforcement, is not the one that represents the best behavior in service, because, although the opening of cracks is adequately controlled by the inclined reinforcement in the reentrant corner, regions were detected where the stresses in the reinforcements were high, namely in the horizontal orthogonal reinforcement and in the distributed orthogonal reinforcement. Therefore, high crack openings in other areas will be expected.

Apparently, in the model with k_d = 1.0, all the reinforcements presents a good behavior in service, however, that may not reflect the possible opening of inclined cracks. Therefore, a sufficiently small inclined reinforcement (0.02cm^2) was included in the model, so as not to contribute to the forwarding of loads, so as to “measure” indirectly the extension in the steel along each load step, as shown in Figure 10.

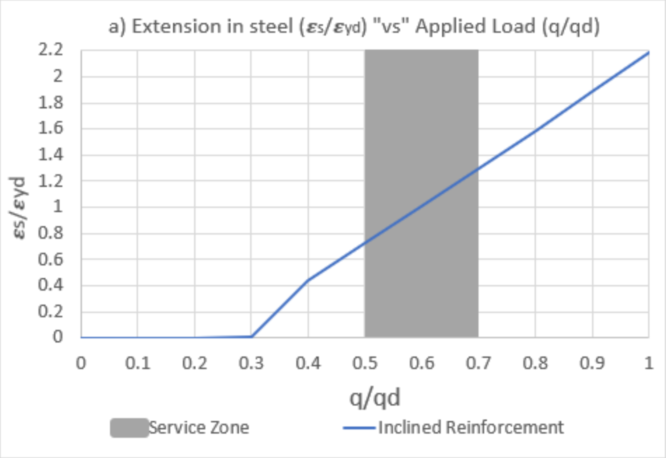

Figure 10 - Extension levels at each load step for the inclined reinforcement (k_d = 1.0).

As far as can be seen, the extension of steel in service is between 0.72ε_{yd} to 1.3ε_{yd}, where ε_{yd} is the yield extension of A500 steel (2.18*10^{-3}). This level of extension is a great indicator that in service there is a large opening of cracks on the concrete in the area of the re-entrant corner and that despite the orthogonal reinforcements having moderate stresses, it is not effectively controlling the crack. Concluding that this model has a bad behavior in service.

Finally, taking into account that intermediate redistributions (between k_d = 0.25 and k_d = 0.5, for example), return intermediate stresses values, it is possible to conclude that values between k_d = 0.35 and k_d = 0.65 also lead to acceptable in-service behavior.

Regarding the reduction of strength of the concrete, it can be seen that its influence on the covering concrete on the upper face of the beam is all the greater the greater the contribution of the orthogonal reinforcement [Figure 11]. As in the experimental tests carried out with dapped end beams, it is common to observe the detachment of the covering concrete close to the final load.

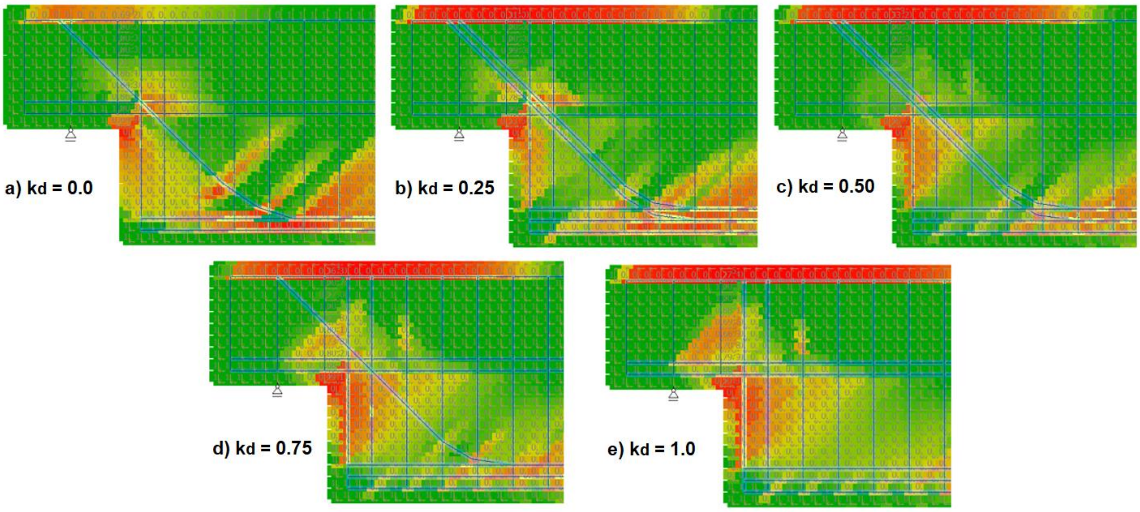

Figure 11 - Reduction of concrete strength (q/qd = 1.0): a) kd = 0; b) kd = 0.25; c) kd = 0.50; d) kd = 0.75; e) kd = 1.0

7. CONCLUSIONS

In the context of this work, using non-linear analysis techniques by the MEF, we seek to study, for the typical situation of the dapped end beam, simply supported and subjected to a concentrated load in the middle span, the different variations that can be considered in the configurations of possible design models, resulting in amounts of reinforcement, orthogonal and inclined, different from those suggested by the reference solution.

As the main criterion for the evaluation and selection of models, the stresses in the reinforcement are evaluated for actions corresponding to the limit state of service, as these are directly related to their behavior in service. As expected, it was possible to confirm that both the orthogonal reinforcements and the inclined reinforcements present higher values of stress in service, the lower the percentage of reinforcement.

Assessing the overall behavior of the structure, by jointly analyzing the stress levels in the reinforcements for the various models considered, it is possible to verify that models that use essentially one of the main equilibrium models, only orthogonal or diagonal reinforcements ($k_d = 0$ and $k_d = 1$), are the ones that perform the most inappropriate.

Thus, through the analyzes carried out, it is possible to conclude that an approximate distribution of forces between the two configurations, guarantees an adequate service behavior, becoming a valid option in the dimensioning of this type of discontinuity regions. This behavior is fully satisfied by the model with $k_d = 0.5$, in which 50% of the load is routed in the orthogonal direction and the rest in the diagonal direction. In this, the losses of strength of the concrete are less intense and less concentrated in certain regions of the structure, thus improving its performance. Finally, redistributions of about 30% of this model also lead to adequate results in service.

BIBLIOGRAPHY

CAMARA, J.; COSTA, A.; ALMEIDA, J.; JÚLIO, E.; ALFAIATE, J. – Estruturas de Betão - Folhas de Apoio às Aulas, Lisboa

CAMARA, J.; ALMEIDA, J.; COSTA, A. – Concrete Structures II: Classes 20 to 25: Stress field models – Discontinuity ("D") regions, Lisboa

EUROCODE 2 – EN 1992-1-1 – Design of concrete structures – Part 1-1: General rules and rules for buildings, December 2004

FALCÓN, J. – Estudio del comportamiento en servicio y rotura de los apoyos a media madera, España, Junio 2015

FERREIRA, M.; ALMEIDA, J., LOURENÇO, M., 2017 – Modelling structural concrete with strut-and-tie model combined with 2D finite elements - a model factor for the assessment of strut-and-tie models, *fib* Symposium 2017, Maastricht, June 12-14 2017

FIB BULLETIN 56 – Model Code 2010, First complete draft - Volume 2, 2010

FIP – Practical Design of Structural Concrete, September 1999

KOSTIC, N. – Computer-Based Development of Stress Fields, 6th International PhD Symposium in Civil Engineering, Zurich, August 23-26, 2006

LOURENÇO, M. – Adaptive Stress Field Models for Structural Concrete, Tese de Doutoramento, Instituto Superior Técnico, Universidade de Lisboa, Fevereiro de 2010

LOURENÇO, M.; ALMEIDA, J. – Adaptive Stress Field Models: Formulation and Validation, ACI STRUCTURAL JOURNAL, Title no.110-S08, January – February 2013

LOURENÇO, M.; ALMEIDA, J. – Adaptive Stress Field Models: Assessment of Design Models, ACI STRUCTURAL JOURNAL, Title no.110-S09, January – February 2013

MUTTONI, A.; SCHWARTZ, J.; THÜRLIMANN, B. – Design of Concrete Structures with Stress Fields, Basel, Boston, Berlin, Birkhauser, 1997

RUIZ, M.; MUTTONI, A. – On Development of Suitable Stress Fields for Structural Concrete, ACI STRUCTURAL JOURNAL, Title no.104-S48, July – August 2007

SCHLAICH, J.; SCHÄFER, K.; JENNEWEIN, M. – Toward a Consistent Design of Structural Concrete, PCI JOURNAL, May-June 1987

SCHLAICH, J.; SCHÄFER, K. – Design and detailing of structural concrete using strut-and-tie models, The Structural Engineer, Volume 69, No.6, 19 March 1991

TJHIN, T.; KUCHMA, D. – Integrated analysis and design tool for the strut-and-tie method, Engineering Structures 29: 3042-3052, 2007

VITONE, A.; PALMISANO, F.; VITONE, C. – Load Path Method (LPM) in Detailing Design, Proceedings of the 2nd International Congress, Session 3 – Modelling and design, Naples, Italy, June 5-8, 2006